RESEARCH DEPARTMENT

FOURIER TRANSFORM GENERATOR

Report No. T-078

(1960/27)

THE BRITISH BROADCASTING CORPORATION
ENGINEERING DIVISION

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SUMMARY

An electronic device is described which, when coupled to a suitable oscilloscope, will display the sine- or cosine-transform of any function capable of being represented at least approximately by not more than forty-one ordinates spaced at equal intervals of the variable. A further limitation is that these ordinates be arranged so that not more than twenty are positioned on each side of the reference point or zero of the variable.

1. INTRODUCTION

Fourier analysis and synthesis is well established as one of the most useful mathematical tools at the disposal of scientists and engineers, whose work deals so often with the properties of wave motion. Unfortunately, the use of the Fourier transform entails laborious computation unless the function to be transformed can be expressed in a closed form leading to readily soluble integrals. It was thought that the existence of a device capable of displaying the Fourier transform (FT) of any function which may be represented by not more than forty-one ordinates, would encourage engineers to undertake the solution of problems which might otherwise be avoided. The device enables the following and similar problems to be solved:

(i) Given a function of time, it is required to find the spectrum which corresponds to it.

(ii) Given the spectrum, it is required to find the function of time which gives rise to it.

The device to be described was originally constructed in order to decrease the time required to carry out the routine Fourier transformations which are necessary for the assessment of optical lenses used in television.[1] The original idea was put forward by G.G. Gouriet. The FT generator as finally constructed has since proved useful in such problems as the convolution of two functions by taking the inverse Fourier transform of the product of the transforms of the two original functions and in obtaining the Hilbert transform of a function.

Fig. 1 is a photograph of the generator, which shows all the operating controls. The function to be transformed is plotted on the front panel by setting the forty-one adjustable markers to represent the ordinates of the function at equally spaced values of the abscissae, the central marker corresponding to the zero value of the abscissae. The sine- or cosine-transform is displayed on the screen of a suitable oscilloscope when the switch marked "SINE/COSINE" is set to the appropriate position.

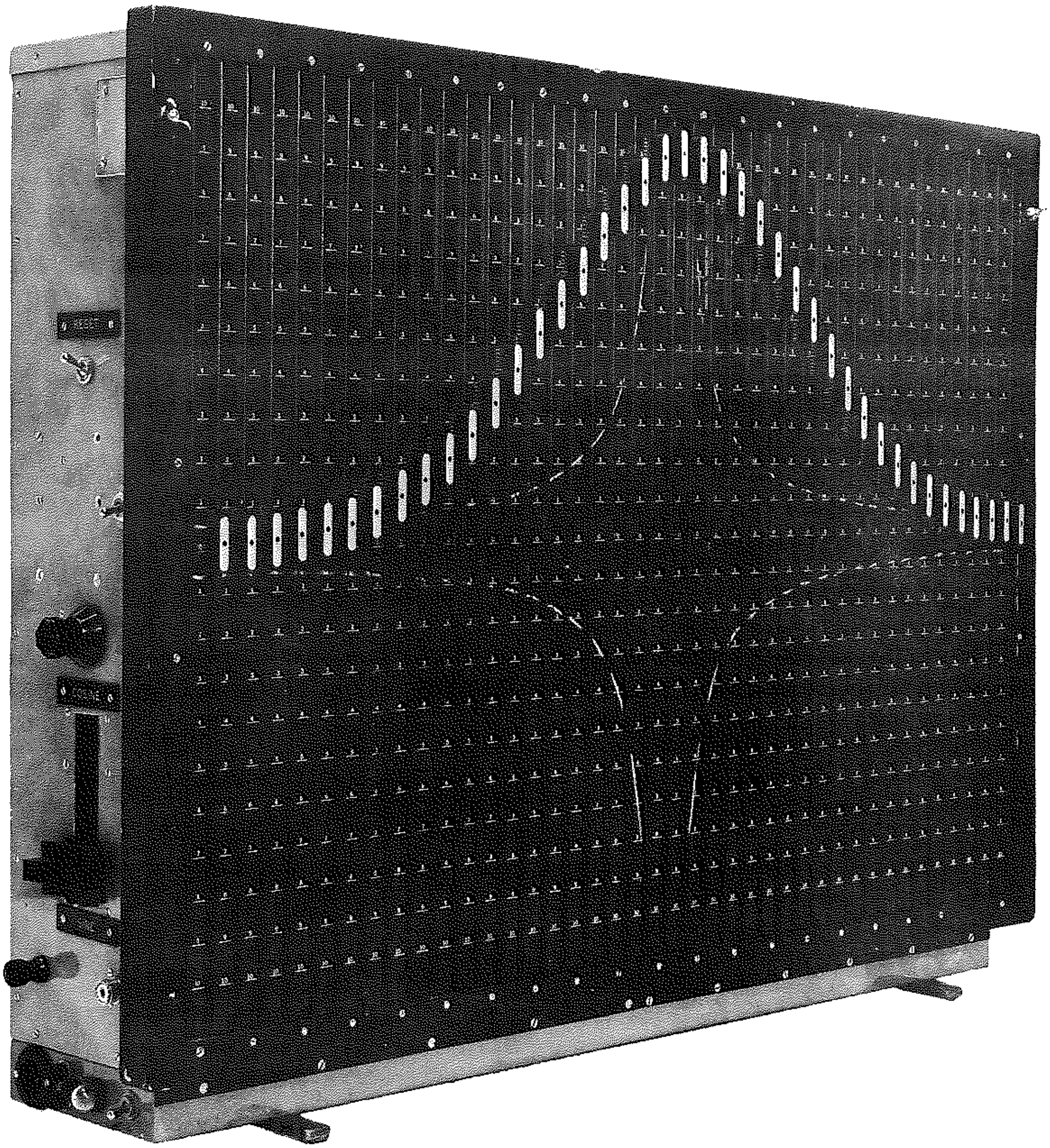

Fig. I - Fourier Transform Generator

The time required to operate the instrument is reduced to that required to plot the original function by means of the adjustable markers, and that required to read and transpose the transform displayed on the oscilloscope screen. The accuracy with which the transform may be obtained is of the order of ±1% and the instrument retains this accuracy over a period of many months without adjustment, because only transistors and passive network components are used in its construction. Less accuracy in the transforms must be expected if the apparatus is used for "unsuitable functions".

2. THEORY AND APPLICATION

2.1. Basic Principles

A "suitable" restricted function of a variable x, extending between the limits $-X$ and $+X$ can be analysed into a harmonic series of sine and cosine functions of the variable x; this series will represent the function correctly within the interval $-X < x < X$. The amplitude of any particular cosine term P_r, or sine term Q_r is given by

$$P_r = \frac{1}{2X} \int_{-X}^{X} f(x) \cos \frac{r \pi x}{X} \, dx \tag{1a}$$

$$Q_r = \frac{1}{2X} \int_{-X}^{X} f(x) \sin \frac{r \pi x}{X} \, dx \tag{1b}$$

Where the sums $(P_r + P_{-r})$ and $(Q_r - Q_{-r})$ represent respectively the amplitudes of cosine and sine components of the rth harmonic. The value of P_r for $r = 0$ is the amplitude of the constant component in the transform of $f(x)$. Numerical methods are sometimes used and these amount to replacing the continuous integrations in (1) by a summation of discrete values of the integrand.

$$P_r = \frac{1}{2n+1} \left(A_{-n} \cos - nr\theta \ \ldots + A_{-1} \cos - r\theta \quad + A_0 + A_1 \cos r\theta \ \ldots + A_n \cos nr\theta\right) \tag{2a}$$

$$Q_r = \frac{1}{2n+1} \left(A_{-n} \sin - nr\theta \ \ldots + A_{-1} \sin - r\theta \quad + 0 + A_1 \sin r\theta \ \ldots \ A_n \sin nr\theta\right) \tag{2b}$$

where $\theta = 2\pi/(2n+1)$ and the A's are the selected ordinates of $f(x)$. Equations (2) apply when the method of sampling is as used in Fig. 2, where there is an odd number of samples and the end ones A_{-n} and A_n are spaced from the limits of the variable by half the normal spacing between ordinates. The effect of altering this arrangement of sampling is dealt with later. Equations (2) can be reduced to

$$P_r = \frac{1}{2n+1} \left[A_0 + (A_1 + A_{-1})\cos r\theta + (A_2 + A_{-2})\cos 2r\theta + \ldots + (A_n + A_{-n})\cos nr\theta\right] \tag{3a}$$

$$Q_r = \frac{1}{2n+1} \left[(A_1 - A_{-1})\sin r\theta + (A_2 - A_{-2})\sin 2r\theta + \ldots + (A_n - A_{-n})\sin nr\theta\right] \tag{3b}$$

In numerical solutions of this type only integral values of r are normally used, but equations (2) and (3) are valid for all values of r although, obviously, when $r\theta$

exceeds 2π the functions P_r and Q_r are repetitive. When the normal sine and cosine tables are replaced by a machine, it is simpler to consider r as a linear and continuous function of time. In other words, if r is replaced by $\omega t/\theta$, $\cos m r\theta$ becomes $\cos m\omega t$. The value of ω can be chosen for convenience of design or instrumentation.

For the cosine transform P_r we require a constant term proportional to A_0 and a set of n harmonic cosine waves; by this we mean that when the lowest-frequency wave is passing through a positive maximum, all the harmonics are also passing through positive maxima. Similarly, for the sine transform, we require a set of n harmonic sinewaves: i.e., all the waves pass through zero with a positive slope at the same time as the lowest-frequency wave does so. If a set of potential dividers is provided, so that the amplitudes of the cosine- or sine-waves can be adjusted to be proportional to the ordinates of $f(x)$ as in equations (2) and (3), and if the cosine waves are added to the constant component, the magnitude of P_r can be obtained for all values of r as a continuous and periodic function of time. Similarly, the summation of the sine-waves will produce Q_r for all values of r. If these output waveforms are displayed on an oscilloscope, the time base can be calibrated in terms of r, and either P_r or Q_r read off for any value of r.

2.2. Principles of Construction and Operation

The unit comprises forty-one potential dividers of which forty control the amplitudes of twenty harmonically related sinusoidal waveforms and the remaining one controls the constant component, which, in fact, is periodically interrupted for the convenience of display. Each potential divider provides positive or negative polarities of its associated ordinate for positions of its slider respectively above or below the central position. The potential dividers consist of straight vertical strips of resistive material, 20 inches (50 cm) long, arranged side by side on the rear of the front panel; the sliders are adjustable from the exterior. Each harmonic wave is supplied to two potential dividers symmetrically disposed about the central divider, which controls the constant term; the lowest-frequency waveform is connected to the two dividers immediately adjacent to the central one. The other pairs are connected to the harmonic waveform sources in such a way that the frequency of the waveform increases as the distance from the centre increases. The sinusoidal waveforms can be made available as cosine- or sine-waves by means of a switch on the side of the instrument case. When this switch is in the "COSINE" position the waveforms from all the sliders are added to form the output transform P_r. When the switch is in the "SINE" position the central potential divider is disconnected and the sum of the waveforms from sliders on the left side of the panel is subtracted from the sum of the waveforms from the right side, as required by equation (3b).

The operational procedure amounts to plotting the function $f(x)$ on the front panel by suitably positioning the adjustable markers, as illustrated in Fig. 1, momentarily depressing a switch labelled "RESET" and, by means of an appropriately labelled switch, selecting either the cosine or the sine transform for display on the associated oscilloscope. The operation of the "RESET" switch has no theoretical significance in the process of obtaining the transform; it is necessary because of the method used to obtain the series of sinusoidal voltages. It must be stressed that the fact that the output functions P_r and Q_r are presented as functions of time in no way restricts the use of the apparatus to functions of time; we have an

analogue device which operates upon a function of any variable x by means of sinusoidal functions of time. Provided that the output display is treated as an analogue and the time base is appropriately calibrated in terms of r, there is no restriction in the dimensions of the quantity represented by x.

Having dealt with the basic operation, there are now two refinements to describe. The output is suppressed for every alternate period of the fundamental frequency, so that a repetitive display on the oscilloscope will include a horizontal line indicating the axis of abscissae. The second refinement consists of the provision of a brilliance-modulation waveform which permits the transform displayed on the oscilloscope to appear as spots located at integral values of r. This is achieved by brightening the oscilloscope display whenever $r = \omega(t - t_0)/\theta$ has increased from $\omega t_0/\theta$ by an integral value. For our purpose t_0 is defined as the instant in time when all the cosine waves are simultaneously at their positive maxima. This could obviously be arranged to occur for various values of θ, that is, for various spacings between the brightened samples, and, in fact, we have taken θ as equal to $2\pi/40$ rather than $2\pi/41$ as used in equation (2). The reason for this choice is not fundamental, but is a practical one connected with the availability of a suitable waveform. This restricts the use of the brightening facility to cases where the function $f(x)$ is divided into forty equal subdivisions of x. If forty-one ordinates are used, two must coincide with the extreme values of x and must both be halved in magnitude. This is dealt with more fully in a later section.

The brightened spot at the centre of the symmetrical display of a cosine transform represents the value of P_0 because the spot occurs at the time when all the cosines are at their positive maxima and their instantaneous sum is proportional to the mean ordinate. The sine waves are all at zero at this instant so the value of Q_0 is always zero. The adjacent spot occurs when the argument or phase of the fundamental has changed by $2\pi/40$ or θ radians, (positive if the spot to the right of the centre is chosen), the second harmonic has changed by 2θ, the third by 3θ; thus when $r = 1{\cdot}0$ the sum of these harmonic waveforms is the (spotted) amplitude of the lowest frequency component of the harmonic series forming the transform. The next spot occurs after a similar time delay and registers the amplitude of the second frequency component of the series. In this way all the amplitudes of the harmonic components of the transform are immediately apparent.

The amplitudes of these components are the amplitudes of the transform at the integral values of "cycles per $2X$" of the variable x. If, for example, the variable x was in fact "time" and the interval $-X$ to X represented, say, two micro-seconds, the transform would consist of amplitudes at integral values of cycles-per-two-microseconds. Thus the horizontal spacing between brightened spots would each represent 500 kilocycles/second. If, for a second example, the variable x was in cycles/second and the interval $-X$ to X represented a frequency difference of one megacycle/second, the horizontal spacing between spots would represent cycles per megacycle/second or microseconds of time.

If $f(x)$ is a periodic function, the transform consists only of the discrete values given by the brightened spots. If $f(x)$ is discontinuous and consists only of the ordinates plotted and is zero at all other values of x, including the values between the ordinates plotted, the transform will be a continuous periodic function of which one complete period is shown on the C.R.T. display. The brightness must be

increased, however, if the smooth curve representing the continuous periodic function is to be seen. In this case the brightness spots can still be used as a calibration of the horizontal axis.

The third case of interest is that where $f(x)$ is a function which is zero for all values of x outside the region $-X$ to X but which is continuous within these limits. The transform for this case is not periodic, but is continuous and is approximated by one complete period of the display with the brightness increased to show the continuous curve. The closeness with which the transform is approximated depends on the accuracy with which $f(x)$ is represented by the ordinates chosen.

2.3. Restrictions Imposed by Sampling

To obtain the most advantage from the transform generator the function $f(x)$ should be divided into forty equal intervals of x and the magnitudes of $f(x)$ at the boundaries of the intervals should be plotted. The extreme end ordinates should be plotted at half magnitude.

The samples will only represent the function $f(x)$ if it contains no Fourier component having a period less than $2\delta x$, where δx is the interval between samples. The discontinuous transforms obtained with the aid of the brightening spots should be accurate up to the nineteenth harmonic component. The twentieth component is sampled twice per cycle; in the case of the cosine transform these samples all coincide with the maxima, so the amplitude will appear double; the samples will all coincide with zeros in the sine transform, so that the twentieth sine component will always be zero, irrespective of the actual amplitude.

The function represented by the series of ordinates can be produced by replacing each ordinate by a continuous function,

$$f_i(x) = A_i \frac{\sin(x-x_i)n\pi/2X}{(x-x_i)n\pi/2X}$$

where n equals the number of the intervals of x into which $f(x)$ is divided and A_i is the magnitude of the ordinate being replaced by $f_i(x)$. These functions will each be centred on the value of x corresponding to the ordinate being replaced and will have zero amplitude at every value of x corresponding to the other ordinates. If the sum of all these functions closely approximates to the function being sampled, the transform will closely approximate to the desired transform.

This can be seen by reference to Fig. 3 which shows, at the top, a function $f(x)$ assumed to consist of a finite sum of $(\sin x)/x$ type waveforms. One of these, $f_0(x)$ is shown centred on the zero value of the variable x. The cosine transform $g_0(\rho)$ of $f_0(x)$ is given by

$$g_0(\rho) = A_0 \int_{-\infty}^{\infty} \frac{\sin \pi x/\delta x}{\pi x/\delta x} \, . \cos(2\pi\rho x) \, . \, dx \tag{4}$$

where δx is the spacing between ordinates along the x axis and A_0 is the amplitude of the ordinate at $x = 0$. The transform $g_0(\rho)$ has a finite value

$$g_0(\rho) = A_0\delta x \text{ for } -1/(2\delta x) < \rho < 1/(2\delta x) \tag{5}$$

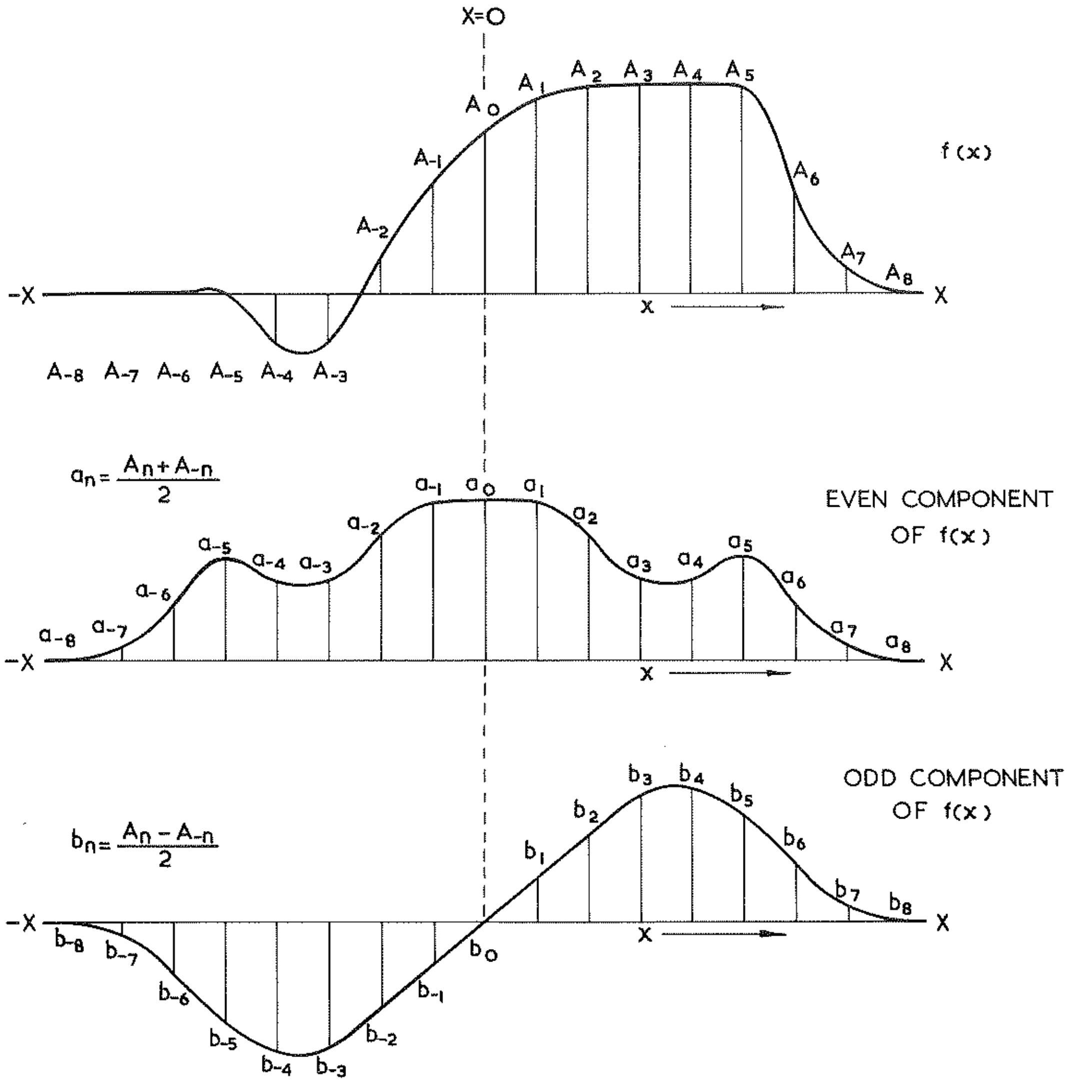

Fig. 2 - An asymmetrical function with its even and odd components

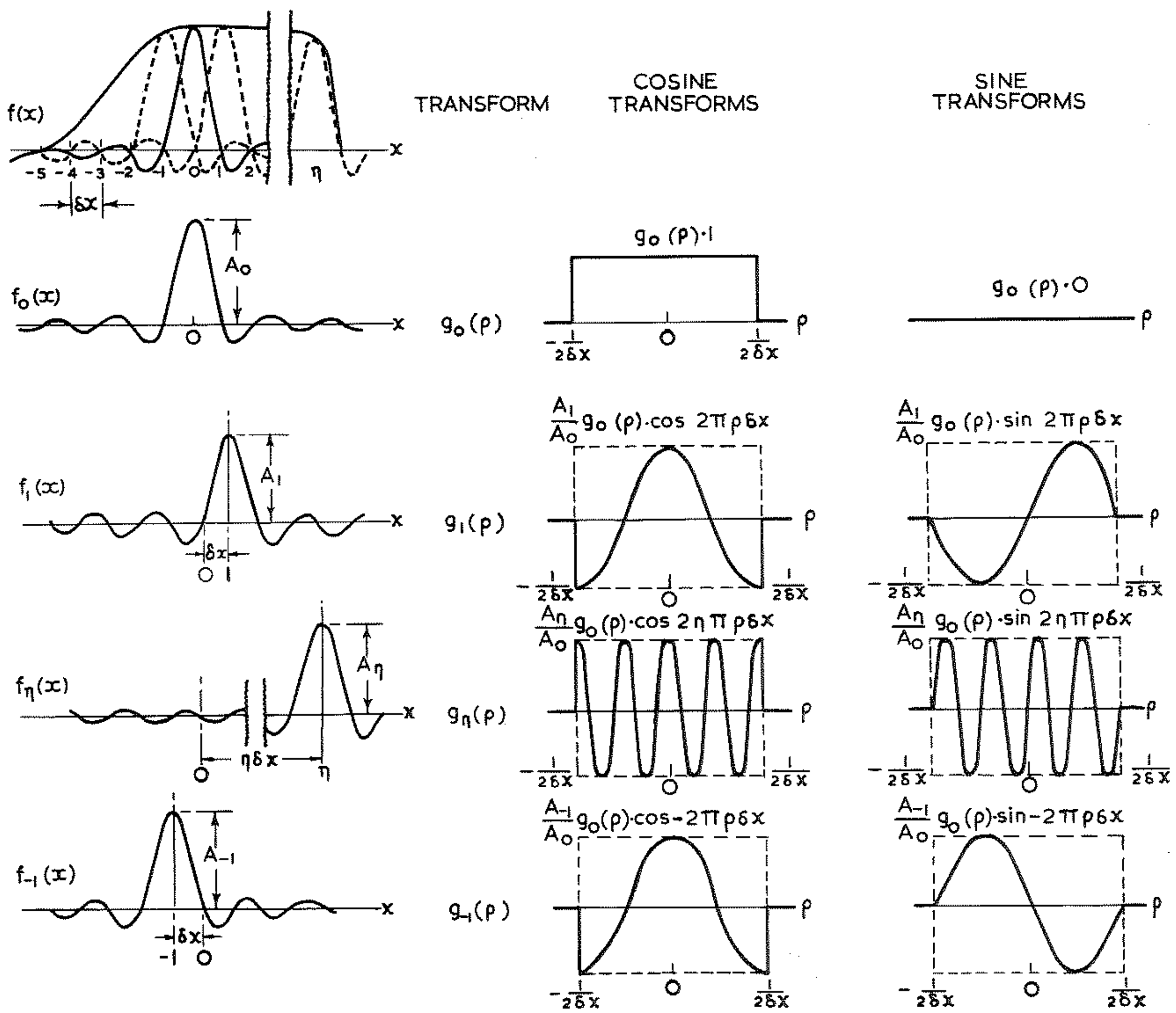

Fig. 3 - Method of sampling using $\frac{\sin x}{x}$ functions

and is zero outside this range. The sine transform is zero. If the $(\sin x)/x$ waveform is displaced from being centred on the zero value of x by an amount $\eta\,\delta x$, the transform can be shown, by means of the Heaviside Shift Theorem, to consist of a cosine transform, $g_0(\rho)\,.\cos 2\eta\pi\rho\delta x$ and a sine transform, $g_0(\rho)\,.\sin 2\eta\pi\rho\delta x$. If the shift is to the left of the zero position, the sign of the amount of shift $\eta\,\delta x$ is considered to be negative, and as shown at the bottom of Fig. 3, the sine transform changes polarity as compared with the transform resulting from a similar shift to the right. It will be seen that the transforms of the samples all consist of a rectangular envelope modulating a cosine or sine wave having a frequency proportional to the distance of the sample from the zero value of the x axis. The rectangular envelope need not modulate each cosine or sine wave individually but can operate on the sum. This, in fact, is the method used in the FT generator.

The assumption that a function can be represented by a finite number of equally spaced $(\sin x)/x$ waveforms implies certain restrictions. First, the function must not contain any Fourier components having a period shorter than twice the spacing between the samples. This means that the transform is restricted to finite limits which, in turn, means that the original function extends to infinity in each direction of the variable x. For a function of infinite duration to be represented by a finite number of samples requires that all other sampling points must coincide with zero amplitude. This requires the function to be oscillatory beyond the limits of the finite samples and to have a period of oscillation exactly equal to twice the sampling interval.

These restrictions will not always be applicable so that the function represented by the samples will, in general, only approximate to the original function.

There is another form of sampling, which, although not incorporated in the device being described, is worthy of some interest. If the function $f(x)$ is represented by ordinates which, as shown in Fig. 4, are each assumed to represent the height of a rectangle having a width equal to δx, the spacing between samples, it is possible for $f(x)$ to be entirely restricted between limiting values of x.

One such rectangle centred on the zero value of x (Fig. 4) can be considered as

$$f_0(x) = A_0\left[\,H\left(x+\frac{\delta x}{2}\right)-H\left(x-\frac{\delta x}{2}\right)\right] \tag{6}$$

where the Heaviside unit step $H(x+\lambda)$ has a value of zero for $x < -\lambda$ and unity for $x > -\lambda$. The cosine transform of this function $f_0(x)$ is

$$g_0(\rho) = A_0\delta x\,\frac{\sin\pi\rho\delta x}{\pi\rho\delta x} \tag{7}$$

The sine transform is zero. As in the previous example, a shift in the position of $f_0(x)$ on the x axis by an amount $\eta\,\delta x$ results in the transform $g_\eta(\rho)$ having the two components

$$\left.\begin{array}{ll} \text{a cosine transform} & A_0\delta x\,\dfrac{\sin\pi\rho\delta x}{\pi\rho\delta x}\,.\cos 2\eta\pi\rho\delta x \\[2ex] \text{a sine transform} & A_0\delta x\,\dfrac{\sin\pi\rho\delta x}{\pi\rho\delta x}\,.\sin 2\eta\pi\rho\delta x \end{array}\right\} \tag{8}$$

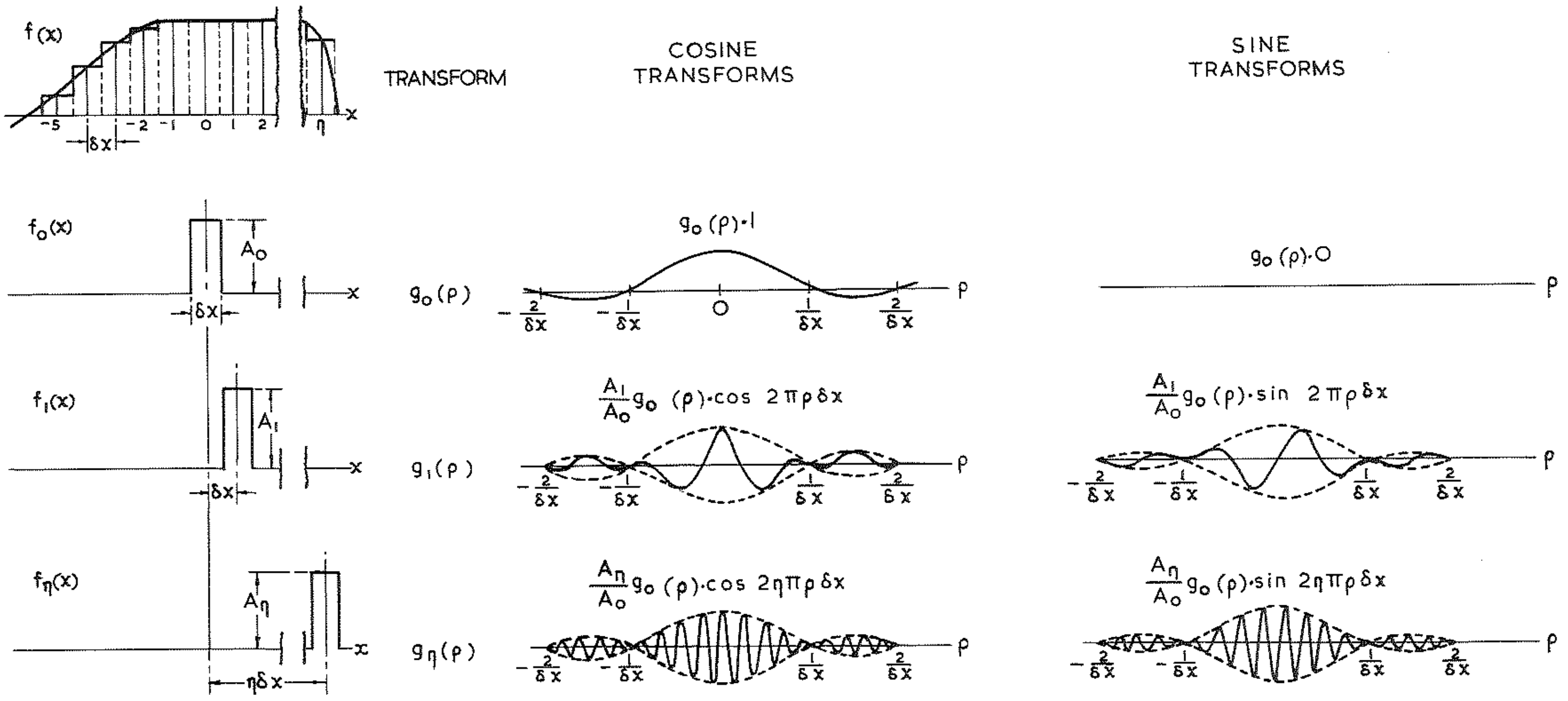

Fig. 4 - Method of sampling using rectangular functions

Again we see that the transforms of the rectangular samples all consist of identical envelopes, of $(\sin x)/x$ shape, each modulating a cosine or sine wave having a frequency proportional to the distance, along the x axis, between the position of the sample and the zero value of x.

The Fourier transform generator could be modified to use this form of sampling by modulating the combined output from the potential dividers with a waveform approximating to

$$\frac{\sin \pi\rho\delta x}{\pi\rho\delta x}$$

between any required limits of x instead of chopping with the square wave as previously described.

If the output from the potential dividers were passed through a suitable modulating stage and, by means of a two-position switch, the appropriate modulation waveform selected, both methods of sampling could be incorporated. In this case, operating the switch would immediately show if the method of sampling were appreciably affecting the transform. It might be necessary to decide which of the two transforms was the more accurate by deciding which of two sampling methods more closely represented the original function.

One further refinement might be worth considering if the method using rectangular samples were incorporated. This concerns the case where the function $f(x)$ has a discontinuity at the zero value of x. Such a case is shown in Fig. 5(a).

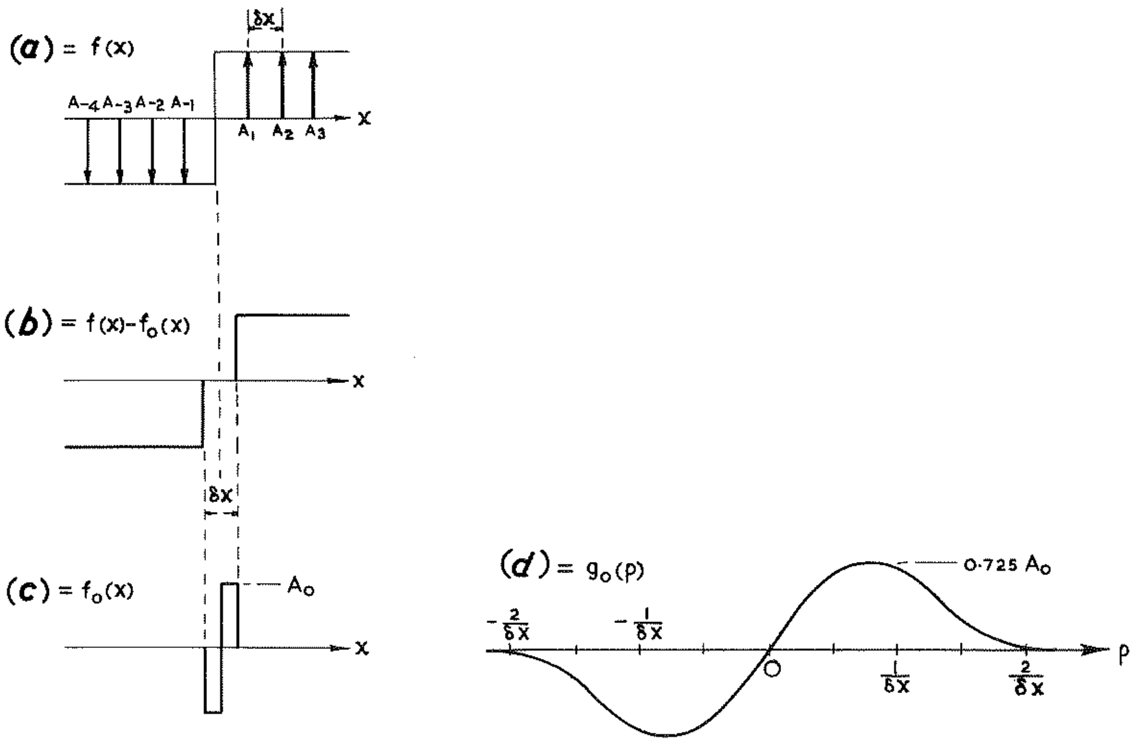

Fig. 5 - Error function caused by sampling at a discontinuity

The ordinate at $x = 0$ has zero magnitude, so that the actual function represented by the rectangular samples is as shown in Fig. 5(b). Ideally, the central ordinate should not be zero, but should be able to represent the sample $f_0(x)$ shown in Fig. 5(c). This sample has the transform $g_0(\rho)$ shown in Fig. 5(d) where,

$$g_0(\rho) = A_0\delta x \frac{\sin^2(\pi\rho\delta x/2)}{(\pi\rho\delta x/2)} \tag{9}$$

The omission of this component from the complete transform may not, in general, be serious because the maximum value of (9) is approximately $0{\cdot}725A_0\delta x$ at $\rho = 0{\cdot}745/\delta x$ and A_0 is only one of forty-one ordinates. If, however, the function $f(x)$ approximates to $1/x$, the error can be very serious and the displayed transform is a triangular wave instead of the correct transform which is a square wave. The error could be eliminated by arranging that, when the generator is switched so as to produce sine transforms, a waveform approximating to the $g_0(\rho)$ of equation (9) is added to the generator output. The amplitude of this waveform could be controlled by the central potential divider, which is not otherwise used in obtaining sine transforms. A very close approximation can be achieved by using the waveform $A_0 \sin(2\pi\rho\delta x/3)$ between the limits $-1/x < \rho < 1/x$. Because this sine wave would have the correct phase only during every third period of the output waveform, it would be necessary to suppress the other two periods.

This further complication may appear to make the method using rectangular samples unnecessarily complicated, but it must be remembered that the function shown in Fig. 5(a) could not be handled by the method using $(\sin x)/x$ samples. It contains Fourier components having too short a period for the finite sampling interval, and must be modified to the "sine-integral" shape characteristic of the step function with a restricted transform. The transform so obtained would, of course, be correct for the function of Fig. 5(a) up to the transform limits imposed, i.e. $\pm 1/2\delta x$.

Fig. 6 summarises diagrammatically the types of function which can be dealt with by the device and the way in which the output waveform should be interpreted. For simplicity, twenty-one ordinates are shown instead of forty-one, also, the functions shown have a point symmetry about the abscissa zero, and so have sine transforms only. Fig. 6(a) shows a discontinuous function consisting only of the ordinate values. In this case the continuous and periodic output waveform is the required transform. If the oscilloscope is arranged to display one complete period of the output waveform this will represent one cycle of the periodic transform. Fig. 6(b) shows a continuous aperiodic function with its transform which is also continuous and aperiodic. This transform can be approximated by one period of the continuous output waveform.

Fig. 6(c) shows a function which, while existing only for discrete values of the variable, x, is also periodic. In this case the transform is periodic and also consists only of discrete values.

Fig. 6(d) shows a continuous and periodic function with a transform consisting only of discrete values as indicated by the brightened spots on one period of the output waveform. The transform is zero everywhere outside the limits enclosing this one period.

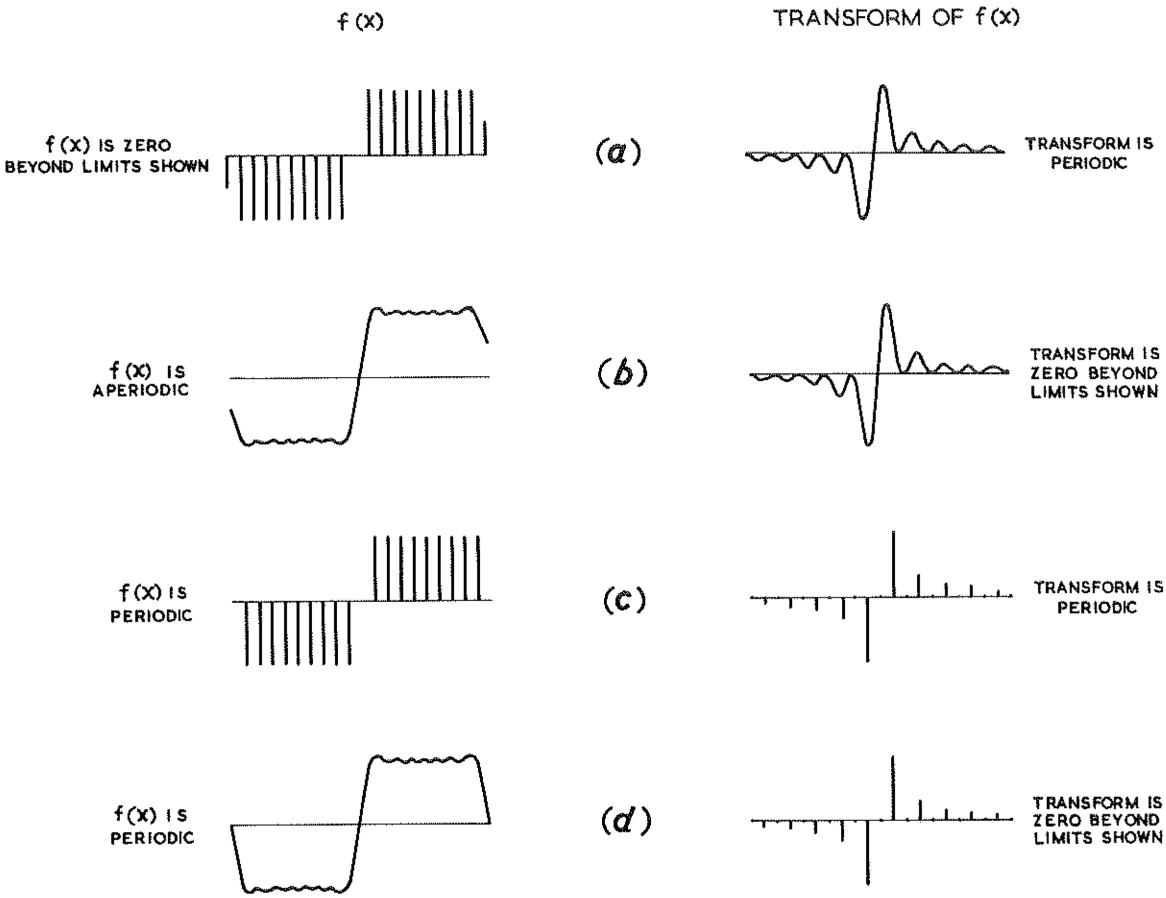

Fig. 6 - Typical functions and their transforms

3. TECHNOLOGY

3.1. Derivation of Harmonic Waves for Synthesis of the Transforms

The two most obvious methods of generating a series of harmonically related waves are either to extract them by means of filters from short impulses recurring regularly at, or below, the required fundamental frequency or, alternatively, to divide down from a frequency much higher than those required. The first method requires considerably more elaborate filters than does the second, because, if each component frequency is to be determined to much better than 1·0%, all the filters must attenuate all unwanted components by considerably more than 40 dB. Filters with this degree of selectivity require careful attention to the stability both of the applied frequency and of their own components in order to avoid changes of relative amplitude and phase with ambient-temperature change.

Using the second method, the harmonics can be obtained in the form of rectangular waveforms by the use of bistable circuits dividing down from higher

frequencies. The filters required to convert these rectangular waveforms to sinusoidal waveforms are relatively simple because there are no unwanted components at frequencies lower than the one required, or less than one octave above it. In the case of a square waveform, the nearest unwanted component is the third harmonic with an amplitude one-third that of the fundamental. Filters capable of such relatively low selectivity can be made to have sufficiently uniform amplitude and phase/frequency characteristics to permit some instability of the master oscillator without introducing relative changes in the amplitudes and phases of the filtered harmonics.

Unfortunately, the second method becomes impracticable if a large number of harmonics is required because the master oscillator must have a frequency equal to the lowest common multiple of the wanted frequencies, and this would necessitate an extremely long chain of divider stages.

In the apparatus described here (see Fig. 7) a modification of the second method was adopted and the master oscillator operates at 120 times the lowest required frequency. This permits ten of the required harmonics to be selected by division. A further two are obtained by selecting the third harmonics of two waveforms from the divider chain and the remaining eight harmonics are produced by intermodulation of waveforms selected from the divider chain. Balanced modulators are used to minimise the unwanted components presented to the filter input. In order to simplify the problem of maintaining constant delay through all the filter chains, simple coupled-circuit band-pass filters are used.

3.2. The Divider Chain

The master oscillator operates at a frequency of 60kc/s giving a fundamental frequency for the unit, henceforward referred to as f_0, of 500 c/s. This frequency was chosen as the lowest compatible with easy attainment of the desired values of "Q" factor in the filter inductances. The oscillator is a transistorised version of the circuit described by Gouriet[2] and is stabilised at a low level to give linear operation. To minimise the overall effects of ambient-temperature changes, the frequency determining elements in the oscillator have been chosen to have the same temperature coefficients as those in the band-pass filters; in practice a long-term stability of about 1 part in 10^4 has been achieved. The output from the oscillator is converted by a blocking oscillator to a series of 60 kc/s pulses which drive the dividing circuits.

For economy in the design of the filtering and multiplying stages, it is desirable that the counters should have push-pull output voltages of high amplitude; for this reason Eccles-Jordan circuits are used for most of the counters. For the highest frequencies, however, where these requirements do not apply, an asymmetrical circuit due to Chaplin and Owens[3] is preferred, being faster, simpler and readily adaptable to function as an automatically re-setting gate. These gating circuits are combined with binary circuits to give division by three and five. Fig. 8 shows the arrangement of the oscillator and counting circuits. Further Eccles-Jordan binaries, not shown, are connected in cascade from the points indicated to produce lower frequencies. It is essential that the various branches of the dividing chain shall operate with a given time relationship. When the unit is switched on, the correct relationship is established by a spring loaded "RESET" switch stopping the oscillator and pulling all the bistables over to the required polarity.

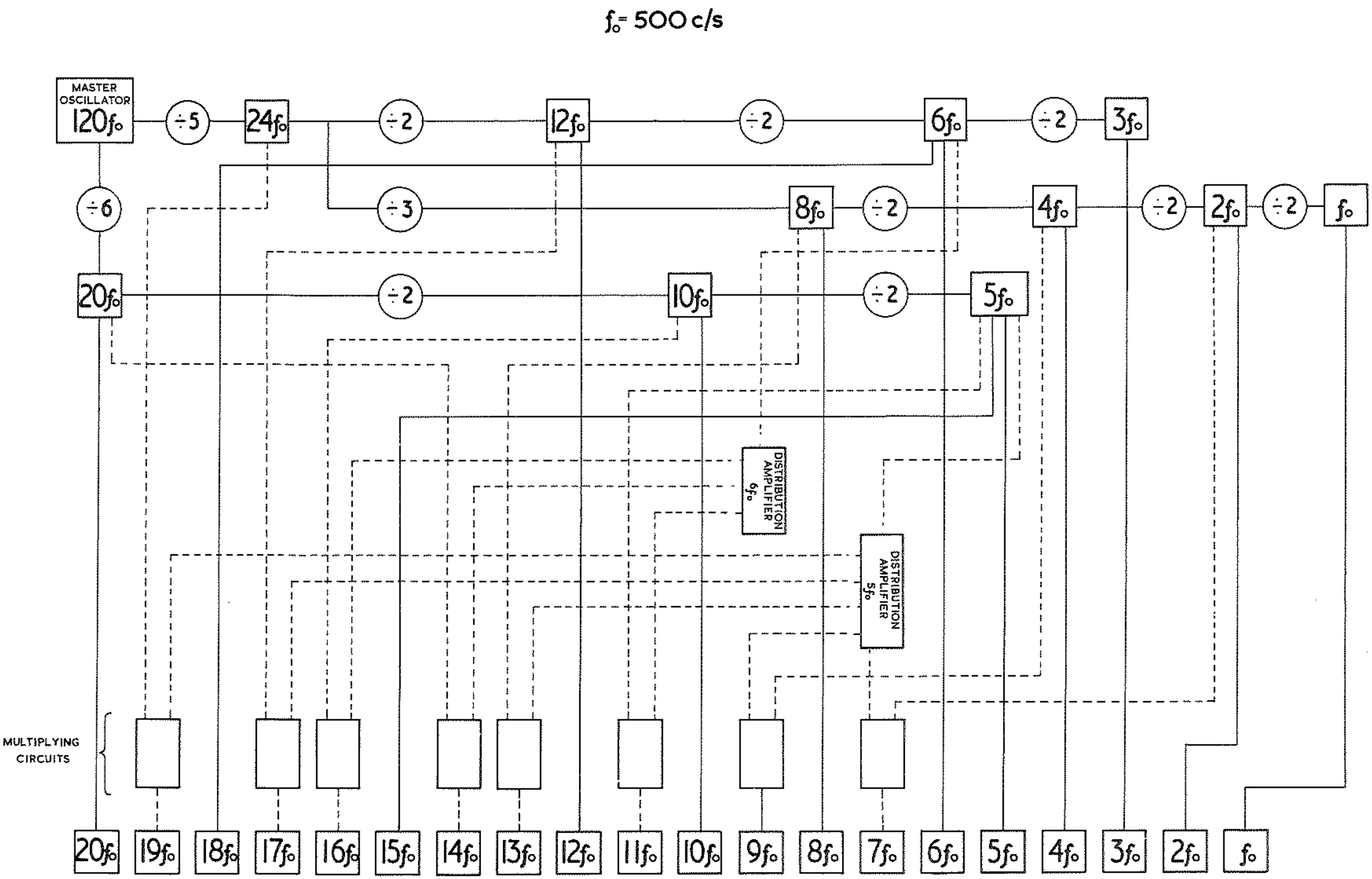

Fig. 7 - Method of frequency derivation

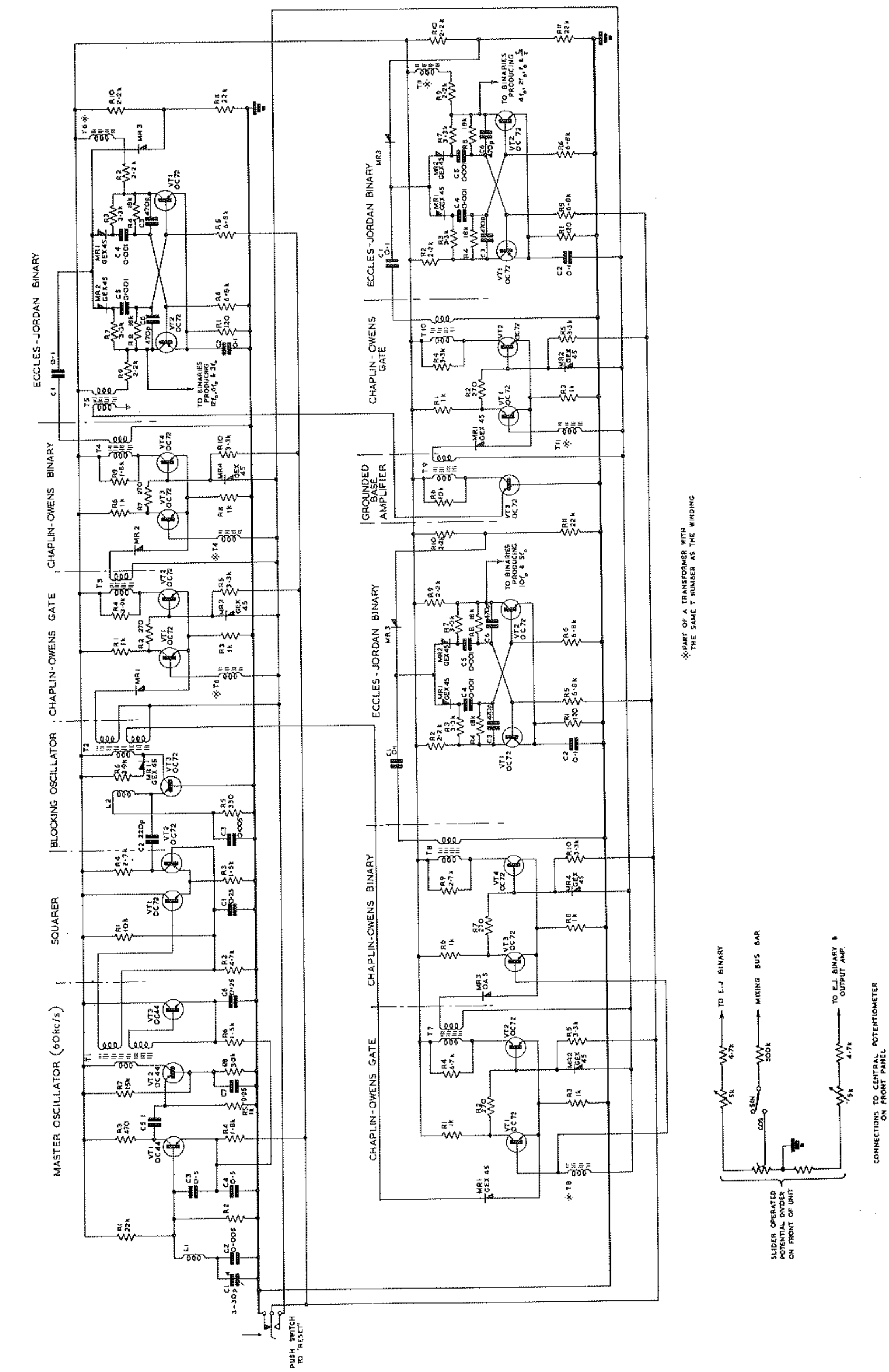

Fig. 8 - Circuits of master oscillator and divider stages

3.3. Filter Circuits

Fig. 9 shows a typical circuit used to filter one of the output harmonics from a divider waveform.

An emitter follower, with provision for inserting quadrature phase shift, drives a common-emitter amplifier having as collector load a capacitively coupled pair of tuned circuits which constitute the band-pass filter. The paraphase outputs of the filter drive a pair of emitter followers with the front-panel potential dividers as emitter loads.

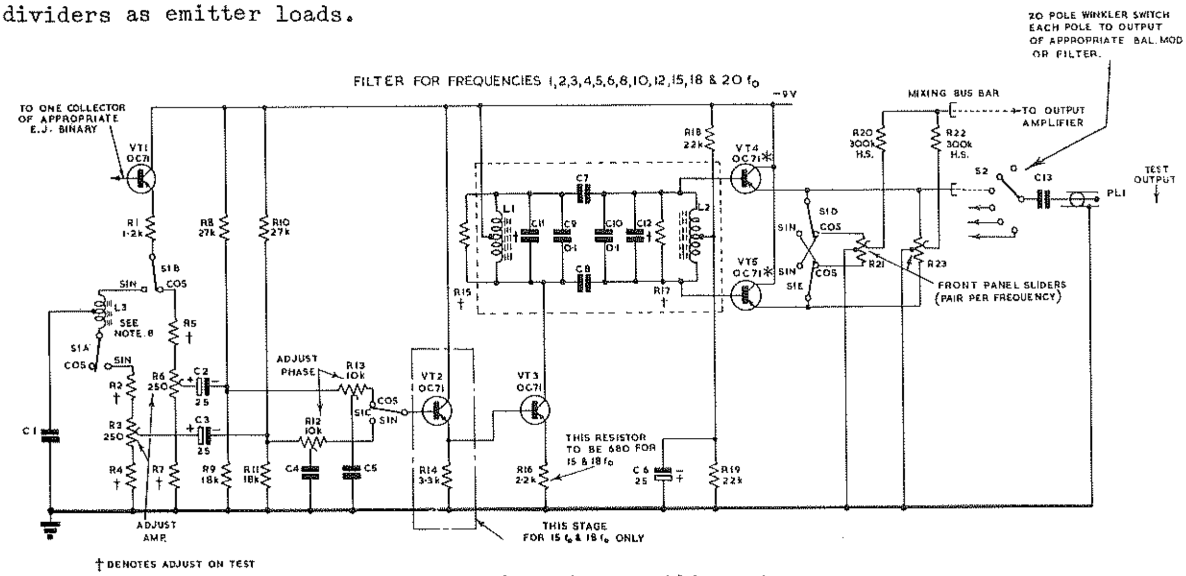

Fig. 9 - Circuit of filter stage

When the binary counter operating at the lowest frequency changes state, all the other counters down the dividing chain also change state almost simultaneously. Because of this, the fundamental components of the square waveforms are approximately in a sine relationship. The filter circuit imposes in all cases a phase shift of $\pi/2$ at its centre frequency, so that the output waveforms are approximately in a cosine relationship. The network L_3C_1 which gives a phase shift of $\pi/2$ at the operating frequency is therefore used in the "SINE" position of the "SINE/COSINE" switch.

The filter configuration used has the advantage of simplicity of design and construction, and of converting the unbalanced input into a balanced output. The two tuned circuits are critically coupled and have a relatively flat amplitude/frequency characteristic around the mid-frequency and a linear phase/frequency characteristic, i.e. constant delay. The value of this delay is made equal for all of the filters so that a small drift in the frequency of the master oscillator does not upset the phase relationship between harmonics. As the filters also use similar components, changes in ambient temperature tend to alter all delays equally, thus minimising relative phase variations. In addition, the inductors and capacitors have approximately-compensating temperature coefficients.

Pre-set controls for the fine adjustment of amplitude and phase, independently variable for sine and cosine outputs, are provided, as shown in Fig. 9, between VT1 and VT2. These controls number eighty in all, so that their associated circuits

are of necessity rudimentary, giving limited control and some degree of interdependence between amplitude and phase. They are, however, adequate to correct for the small drifts encountered once the device has been initially set up. For this initial alignment small capacitors (C11 and C12) and selected resistors (R2, R4, R5 and R7) are used to bring phase and amplitude respectively within range of the controls.

Two frequencies, $15f_0$ and $18f_0$, are derived by filtering the third harmonic from a divider waveform. This necessitates additional gain, which is obtained by lowering the emitter resistor of VT3 and by adding a further emitter follower to preserve the high impedance offered to the fine-adjustment circuits.

3.4. Multiplying Circuits

The type of circuit used to derive the remaining eight frequencies is shown in Fig. 10; in every case one or other of the waveforms at frequencies $5f_0$ or $6f_0$ is mixed in a balanced modulator circuit with an output from the dividing chain. Emitter followers distribute these two waveforms, unfiltered to avoid the introduction of extra delay, which would upset the "equal delay" conditions previously mentioned.

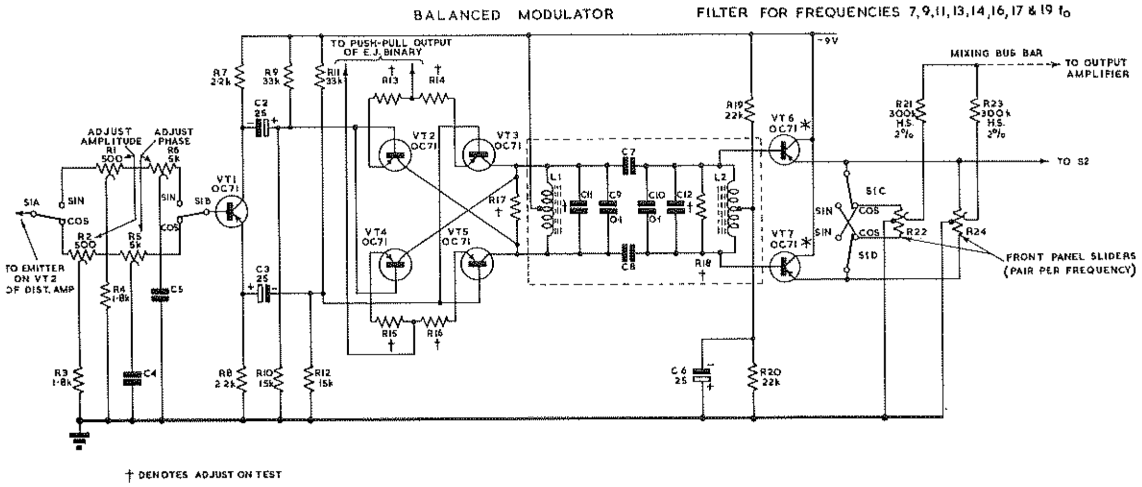

Fig. 10 - Circuit of balanced modulator

The outputs of the multiplying circuits are basically in a "sine" relationship, and quadrature phase shift must therefore be introduced when the switch is in the "COSINE" position. This is conveniently done by transferring to the stages distributing the waveforms at frequencies $5f_0$ and $6f_0$ the same networks which are used in the "SINE" position of the switch.

The multiplying circuit consists of the four transistors VT2-VT5 connected as a balanced modulator. The two states of the associated binary circuit cause the pairs VT2, VT3 and VT4, VT5 to be alternately cut off and biased to conditions of linear amplification. Push-pull signals containing the frequency $5f_0$ or $6f_0$ are applied to the bases, paired differently, while the third possible pairing arrangement is used to couple the four collectors to the input of the filter. Beyond this point the circuit is identical with that used for the directly filtered frequencies.

3.5. Zero Frequency Component

The output from the central potential divider contributes the "constant amplitude" component of the transform and should therefore carry a direct voltage. However, since this would necessitate the use of d.c. coupling through the output amplifier and the oscilloscope, the constant amplitude component is included in the signal by a modulation method which does not involve the handling of d.c. The central potential divider carries a square wave of frequency $f_0/2$ and amplitude equal to the peak amplitude of the sinusoidal voltages across the other dividers. The required constant-amplitude component is thereby added to and subtracted from the signal during alternate periods of $1/f_0$. The output amplifier then suppresses the resultant signal to zero during the "subtraction" periods, so that the final output consists of alternate periods of the transform, with its constant-amplitude component, and of a correctly positioned horizontal axis. By suitably synchronizing the time base of the display oscilloscope, the two may be superimposed.

3.6. Output Amplifier

The outputs from the forty-one front panel potential dividers are combined by connecting each slider, through a 300,000 ohm resistor, to a common bus-bar. The departure from linearity due to this degree of loading has a maximum value of 0·5% of the full voltage across the potential divider.

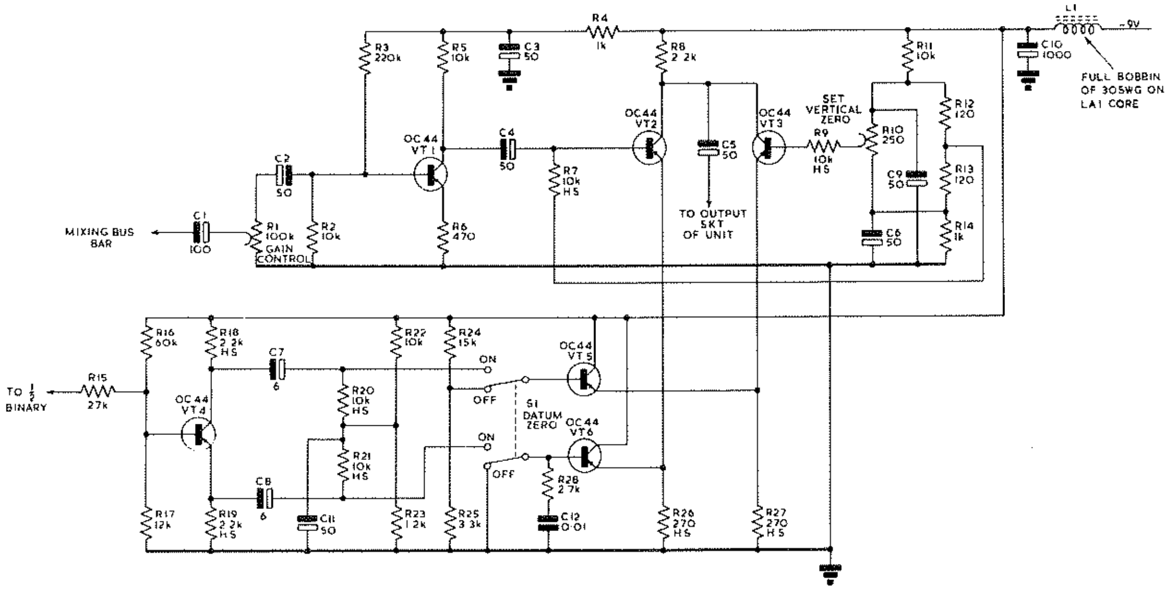

Fig. 11 - Circuit of output amplifier

Fig. 11 shows the output amplifier to which the combined signal from the mixing bus-bar is taken. It is a straightforward two-stage common-emitter amplifier, except that for alternate periods of $1/f_0$ the output transistor VT2 is cut off and a transistor VT3 is switched on. VT3 carries the same d.c. as VT2, but no signal, and shares the same collector load.

3.7. Power Supply

The unit is powered by a conventional stabilised power supply giving an output of -9 volts.

3.8. Performance of the Experimental Model

The potential dividers on the front panel of the generator are formed by strips of Paxolin with a carbon deposit on one surface. The carbon strip is about $\frac{1}{2}$ in. (1·3 cm) wide and 20 in. (50 cm) long and has a resistivity of 500 ohms per square unit. The centre tap is made by means of a metallic deposit between the carbon and the Paxolin. This deposit forms a strip $\frac{1}{8}$ in. (0·3 cm) wide across the full width of the carbon. The sliding contact is made by means of a roller.

It was initially feared that these potential dividers would have short lives, but, as no more suitable material was known to the authors, it was decided to make them easily replaceable. In fact, although the unit has been in service for over one year, no trouble has been caused by wear of the carbon tracks which have not, therefore, been replaced.

It was originally intended to measure the calibration error of each potential divider and to scrape away some of the carbon deposit, where necessary, to improve the accuracy of the scale over the entire range of dividers. This has not been considered necessary on the experimental model, as the overall accuracy has proved sufficiently good for the purpose, without this refinement.

The stability of the circuits appears very satisfactory apart from a slight drift in the vertical direction of the position of the horizontal reference axis during the first few minutes after switching on. This fault would, of course, be serious if not noticed, but it is immediately obvious if the amplitude control is turned to zero and a double trace is observed on the C.R.T. An adjustment control is provided to correct this error. In subsequent models it is hoped that the fault will be removed by improved d.c. stabilisation of the two transistors VT2 and VT3.

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